Feedback Control of a Cylinder Wake Low-Dimensional Model

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Nomenclature

A_k = time-dependent coefficient of the kth proper orthogonal decomposition mode
A_k^{est} = estimation of A_k
B = matrix containing the b_k coefficients
b_k = coefficients associated with the control input
c_{ij}^k = coefficients of the nonlinear function g_k
f_a = control input resulting form external forcing of the cylinder
g_k = nonlinear function of the time-dependent low-dimensional modes
J = Jacobian of the open-loop low-dimensional model
J_c = closed-loop Jacobian
K_p = proportional gain of the linear feedback controller

Introduction

TWO-DIMENSIONAL bluff-body wakes have been investigated for quite some time. In a two-dimensional cylinder wake, self-excited oscillations in the form of periodic shedding of vortices are observed above a critical Reynolds number of around 50 (Ref. 1). This behavior is referred to as the von Kármán vortex street. These flow-induced nonlinear oscillations lead to some undesirable effects associated with unsteady pressures such as fluid–structure interactions[2] and lift/drag fluctuations.[3] Also, the vortices themselves greatly increase the drag of the bluff body, compared to the steady wake that can be observed at lower Reynolds numbers Re. Monkewitz[4] showed that this von Kármán vortex street is the result of an absolute, global instability in the near wake of the bluff body. Farther downstream the flow is convectively unstable.

Many attempts to improve the unsteady vortex street have been made. When active open-loop forcing of the wake is employed, the vortices in the wake can be locked in phase to the forcing signal.[5] Whereas these findings suggest that the dominant structures in the flowfield can be influenced by the forcing, it also strengthens the vortices and, consequently, increases the drag. Different forcing methods are effective in influencing the behavior of the flow. Acoustic excitation of the wake, longitudinal, lateral, or rotational vibration of the cylinder model, and alternate blowing and suction at separation points[5] have been used. None of these open-loop forcing methods have been shown to reduce the drag, independent of frequency and amplitude employed. The only way of suppressing the self-excited flow oscillations is by the incorporation of active closed-loop flow control.[6] Experimental studies show that a linear proportional feedback control based on a single-sensor feedback is able to delay the onset of the wake instability, rendering the wake stable at Reynolds numbers about 20% higher than the unforced case. Above $Re = 60$, a single-sensor feedback may suppress the original mode but destabilizes one of the other modes.[2] Therefore, better control strategies are needed to stabilize the wake at Reynolds numbers of technical interest.

A more advanced control approach was employed by Gillies.[7] He used the proper orthogonal decomposition (POD) technique to obtain a low-dimensional model of the flowfield. In his simulation, a one-step temporal neural predictor network performed multisensor evaluation and nonlinear prediction of the four most dominant POD mode amplitudes in the wake. A neural network controller was then used to determine the actuator command based on the estimated mode amplitudes. Using this approach he was able to stabilize the flowfield at a Reynolds number of 100.

Whereas Gillies's controller was successful in a simulation, a simpler controller better suited for real-time implementation is needed to be able to control a real-life flowfield in an experiment. The control scheme presented in this Note uses the same POD technique as by Gillies[7] but investigates the wake stability of a linear feedback controller acting on the estimation of a single-POD mode only. Also, the controllability of the wake model is examined as a necessary condition before applying the described controller.

POD-Based Flow Model

To be able to run simulations to evaluate feedback control algorithms, a simple model of the flowfield was sought. Whereas the model primarily needs to capture accurately the dynamic behavior of the flowfield, it also needs to model the effect of the actuator on the flow. Gillies[7] developed such a model. His starting point was the result of a direct Navier–Stokes simulation of the two-dimensional circular cylinder wake at a Reynolds number of 100. Based on the transient data from this simulation, he developed a POD representation of the modes and mode amplitudes of the flow. Because POD is an optimal approach, in that it will capture a large amount of the flow energy in the fewest modes of any decomposition of the flow,[8] the model can be truncated and still accurately represent the flowfield. Gillies[7] found that four modes suffice for simulating the cylinder wake at the given Reynolds number.

After obtaining the spatial mode shapes as well as the temporal coefficients, a low-order system of partial differential equations was developed. This system of equations accurately represents the dynamic behavior of the flowfield[6] and is of the form

$$\dot{A}_k = g_k(A_1, A_2, \ldots, A_M) + b_k f_a \quad \text{for} \quad k = 1, 2, \ldots, M \tag{1}$$

The function g_k is chosen to be cubic (as presented in Appendix A of Ref. 6):

$$g_k = c_0^k + c_{1i}^k A_i + c_{2ij}^k A_i A_j + c_{3ijl}^k A_i A_j A_l \qquad \text{for} \quad i, j, k, l = 1, 2, \ldots, M \tag{2}$$

where A_k is the time-dependent coefficient of the kth mode, M is the total number of modes in the low-dimensional model, g_k is the nonlinear function of the time-dependent coefficients, and f_a is the control input (arbitrary forcing) to the cylinder. The coefficients in Eq. (2) are obtained from the temporal POD coefficients using a least-squares fit.

Control Approach

For real-time implementation of closed-loop control, based on a POD low-dimensional model, it is worthwhile to develop estimator and controller strategies that are simple yet effective, thereby avoiding unnecessary time delays. The controller is based on the estimation of the temporal coefficients of the POD states. It may be possible to keep the amount of real-time sensing and processing of data at a minimum if only one mode is required to be estimated by the controller. Let the control law be as follows:

$$f_a = -K_p \cdot A_1^{\text{est}} \tag{3}$$

Received 17 July 2002; revision received 24 February 2003; accepted for publication 4 March 2003.

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where K_p is the proportional gain of the P controller and A_1^{est} is the estimate of the time-dependent coefficient of mode 1, that is, A_1.

In an experiment, A_1^{est} needs to be extracted from a number of sensors placed in the wake. Recently, Siegel et al.[9] developed an accurate estimator for the first four POD modes of a circular cylinder wake based on five sensors in a real flowfield. A linear stochastic estimator was applied to water tunnel particle image velocimetry data. Based on this experience,[9] in this effort estimation errors are neglected, and it is assumed that $A_1 \sim A_1^{\text{est}}$.

The questions addressed in the next section are the following. Is stability of all POD states assured for such a controller? How are the gains of the controller determined? Is such an approach effective?

Controllability and Stability of a POD Model

Let us examine Eq. (1). For the origin, $A_k = 0$ as the desired equilibrium or fixed point, the aim is to find an appropriate control law for f_a that will make the equilibrium stable. The current effort only concentrates on stability issues. The function g_k can be expanded locally as a Taylor series about the desired equilibrium point:

$$\dot{A}_k = g_k(0) + \frac{\partial g_k(0)}{\partial A_j} \cdot A_j + \mathcal{O}(|A_k|) + b_k \cdot f_a \tag{4}$$

where $\mathcal{O}(|A_k|)$ represents higher-order terms of the expansion and can be neglected for the stability analysis. For the wake POD model, we observe $g_k(0) \sim 0$. Therefore, the linearization of $\dot{A}_k$ about the desired equilibrium point is

$$\dot{A}_k = J \cdot A_j + b_k \cdot f_a$$

where the Jacobian J is

$$J = \begin{bmatrix} \dfrac{\partial g_1(0)}{\partial A_1} & \cdots & \dfrac{\partial g_1(0)}{\partial A_M} \\ \cdots & \cdots & \cdots \\ \dfrac{\partial g_M(0)}{\partial A_1} & \cdots & \dfrac{\partial g_M(0)}{\partial A_M} \end{bmatrix} \tag{5}$$

Inserting the control law in Eq. (3) into the linearization of $\dot{A}_k$ yields

$$\dot{A}_k = J_c \cdot A_j$$

where

$$J_c = \begin{bmatrix} \dfrac{\partial g_1(0)}{\partial A_1} - b_1 \cdot K_P & \cdots & \dfrac{\partial g_1(0)}{\partial A_M} \\ \cdots & \cdots & \cdots \\ \dfrac{\partial g_M(0)}{\partial A_1} - b_M \cdot K_P & \cdots & \dfrac{\partial g_M(0)}{\partial A_M} \end{bmatrix} \tag{6}$$

J_c is the closed-loop Jacobian, and a linear stability analysis based on J_c will provide an insight into the behavior of the closed-loop system.

For the nonlinear system given in Eq. (1), the simplest approach to study controllability is to consider its linearization[10] as described in Eq. (5).

The pair (J, B) is defined to be state controllable if and only if there exists a control f_a that will transfer any initial state A_k $(t = 0)$ to the desired equilibrium point in finite time.

Based on the work of Nijmeijer and van der Schaft,[10] Eq. (5) and the following algebraic condition for controllability are written for $B = [b_1, b_2, \ldots, b_M]$:

$$\text{rank}\Big(B \ \vdots \ JB \ \vdots \ J^2B \ \vdots \ \cdots \ \vdots \ J^{n-1}B\Big) = n \tag{7}$$

The conditions for asymptotic stability can now to stated, based on those proposed by Glendinning[11] for linearized models about their equilibrium point, as follows: For the linearization given in Eq. (6) if the Jacobian J_c, has n eigenvalues, each of which has a strictly negative real part, then the equilibrium point is asymptotically stable.

In addition to conditions for stability, it is also important to make sure that the closed-loop linearized system is hyperbolic. A fixed point of an nth-order system is hyperbolic if all of the eigenvalues of the linearization (Jacobian) lie off the imaginary axis. The Hartman–Grobman theorem states that the local phase portrait near a hyperbolic fixed point is "topologically equivalent" to the phase portrait of the linearization; in particular the stability type of the fixed point is faithfully captured by the linearization (see Ref. 12).

A linearized system [see Eq. (5)] that is hyperbolic is equivalent in terms of stability and bifurcations, chaos and attractors, equilibria and limit cycles to the nonlinear POD model [see Eq. (1)]. From a practical point of view, it is the aim of the control design to find an appropriate gain K_p that will render all of the eigenvalues of J_c to have a negative real part. In addition, the eigenvalues need to lie off the imaginary axis by an adequate margin so that the system is hyperbolic.

Results and Discussion

The proposed control strategy is applied to the POD model. This nonlinear four-mode prototype model contains 72 coefficients in all. The matrices J and B of Eq. (7) are extracted from the model to examine whether the linearized system is controllable:

$$\text{rank}\Big(B \ \vdots \ JB \ \vdots \ J^2B \ \vdots \ \cdots \ \vdots \ J^{n-1}B\Big) = 4$$

where

$$J = \begin{bmatrix} 0.0065 & 0.1447 & 0.0518 & -0.0258 \\ -0.1293 & 0.0065 & 0.0060 & 0.0364 \\ -0.0008 & -0.0000 & -0.0347 & -0.3156 \\ 0.0003 & 0.0007 & 0.2433 & -0.0292 \end{bmatrix}$$

$$B = \begin{bmatrix} -9.998e{-}03 \\ -9.866e{-}03 \\ -6.0577e{-}03 \\ -6.632e{-}03 \end{bmatrix} \tag{8}$$

The controllability matrix for the four model prototype model yields a rank of four. Therefore, the pair (J, B) is state controllable. The eigenvalues of the open-loop Jacobian J are $0.0065 + 0.1367i$, $0.0065 - 0.1367i$, $-0.0320 + 0.2772i$, and $-0.0320 - 0.2772i$.

Not all of the eigenvalues of J have a negative real part. Therefore, the open-loop system is unstable at the desired equilibrium point $(A_k = 0)$. The nonlinear prototype wake model developed by Gillies[6] was simulated using the MATLAB® ODE45 solver for nonstiff differential equations, medium-order method. The results obtained for the open-loop simulations compare well with those provided by Gillies[6] for the unforced prototype modes. (See the uncontrolled portion of Fig. 1.) For the closed-loop simulations, an appropriate gain K_p is sought that will render all of the eigenvalues of the closed-loop Jacobian J_c to have a negative real part. For $K_p < 1.6$, there is a pair of eigenvalues that have positive real parts. At $K_p = 3$ an effective response is observed as shown in Fig. 1.

In the current investigation, the POD mode amplitudes are made directly available as input to the controller. For meaningful applications, these can be readily estimated from sensors placed in the flowfield to implement this control strategy in an experiment or a direct numerical simulation. In recent work,[9] the effectiveness of a linear stochastic estimation (LSE) procedure for real-time estimation of POD mode amplitudes was demonstrated. Using five sensors, an estimate of mode 1 with an error in the order of a few percent was achieved. Because LSE does not involve any iterative procedure to estimate the mode amplitudes at each time step, it is very suitable for implementation on a real-time processor to enable feedback based on mode amplitudes in an experiment.

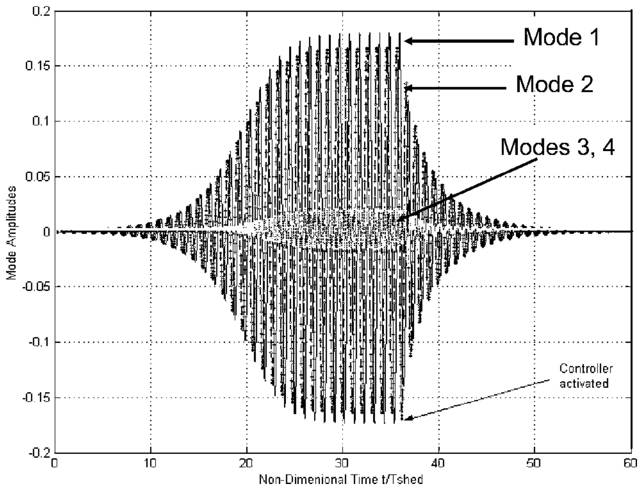

Fig. 1 Flow response to proportional feedback, $K_p = 3.0$.

Conclusions

A simple approach to control the von Kármán vortex street behind a two-dimensional circular cylinder based on the proportional feedback of the estimate of just the first POD mode has been developed. A stability analysis of this control law was conducted after linearization about the desired equilibrium point (origin) and conditions for controllability and asymptotic stability were developed. The control approach, applied to the four-mode cylinder wake POD model at a Reynolds number of 100 stabilizes the wake for a proportional gain above 1.6. Whereas the controller uses only the estimated amplitude of the first mode, all four modes are stabilized. This suggests that the higher-order modes are caused by a secondary instability. Thus, they are suppressed once the primary instability is controlled. The control approach will be further examined to observe its sensitivity to time delays, actuator limitations, modeling and estimation errors, and sensor noise.

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A Mechanism for Flame Acceleration in Narrow Tubes

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Introduction

THIS Note describes a process by which a low-speed, laminar flame and the laminar boundary layer created by this flame interact dynamically to increase the flame propagation velocity exponentially.[1,2] The condition for this flame acceleration is that the flame propagates in a narrow enough channel with heated walls. This phenomenon has potential applications to micropropulsion, and it may describe aspects of explosion initiation in fissures in condensed-phase energetic materials and deflagration-to-detonation transition.

The key element is the presence of the boundary layer that develops along the walls ahead of the flame. This boundary layer develops as the flame propagates, and it is due to fluid motion in unburned material that is induced by expansion of the heated material at the flame front. The fluid motion that develops in the unreacted material ahead of the flame is caused by the expansion of the gas as it burns and acts like a piston. If the flame is accelerating, it generates pressure waves; if it is decelerating, it generates expansion waves. (There may also be a relatively smaller boundary layer behind a weak shock that could arise as a spark lights the flame, but this is much smaller than the boundary layer developing as the flame itself propagates and generates fluid motion ahead of it.)

Figure 1 shows the basic geometry: a flame propagating from the closed end toward the open end of a channel. There is a considerable body of work in the combustion literature on flow instabilities and how they affect the evolution of flames in tubes. There is the relatively early work on this geometrical configuration, for example, Refs. 3–5, in which flames were ignited by sparks and propagated for a time as laminar flames before becoming turbulent. In some cases, the flames oscillated and even reversed direction for a time before resuming movement toward the open end of the channel. There are recent measurements reported on flame propagation in narrow channels (less than 1 cm height) in which the flame accelerated and there was a transition to detonation (G. O. Thomas, private communication). In other recent experimental studies[6,7] with configurations related to Fig. 1, the width, length, and mixture composition of the gases were varied and the observed oscillation frequency was related to the acoustic properties of the channel. Related theoretical and

Received 1 November 2002; revision received 11 March 2003; accepted for publication 11 March 2003.

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